As a model of giant flares in magnetars, we carried out relativistic force-free simulations of twist injection into magnetic loops at the surface of a magnetar. As the magnetic energy is accumulated inside the arcades, they expand with relativistic speed. We found that strong electric fields are induced ahead of the expanding magnetic arcade. Particles can be accelerated in this region, in current sheets formed inside the expanding loops, and by electric potential difference along the loops. We carried out 2-dimensional Particle-In-Cell (PIC) simulations to study the particle acceleration in these regions. Particles accelerated inside the loops create hard X-ray emitting thermal plasma.

§1. Introduction

Soft Gamma-ray Repeaters (SGRs) are one of the most energetic phenomena in our galaxy. In SGR 1806-20 on 2005 December 27, its peak energy was $10^{46}$ erg (Terasawa et al. 2005). Its origin is attributed to a neutron star with a strong magnetic field ($\approx 10^{15}$G). Thompson et al. (2001) proposed that this emission is caused by the dissipation of a non-potential magnetic field (Thompson and Duncan 2001, Thompson, Lyutikov & Kulkarni 2002). When magnetic twist is accumulated inside a magnetar, Lorentz force leads to a crustal motion, which injects magnetic twists from inside a magnetar into the magnetosphere. A sudden reconfiguration of a magnetic field may produce flares (Lyutikov 2003). Lyutikov (2006) proposed that SGRs eject relativistic, strongly magnetized, weakly baryon-loaded magnetic clouds. By this dissipation processes, the large amount of magnetic energy is converted into kinetic energy.

At the surface of a magnetar, since magnetic field is so strong that we can ignore plasma inertia ($B^2/8\pi \gg nm_e c^2$) and plasma pressure. Thus, force-free condition will be satisfied.

Force-free simulations has been applied to study pulsar magnetosphere (Komissarov 2002, Asano et al 2005, Spitkovsky 2006). In section 2, we present a result of relativistic force-free simulations of the dynamics of magnetic loops in SGRs. We also carried out 2-dimensional Particle-In-Cell (PIC) simulations to study the particle accelerations in relativistically expanding loops.
§2. Relativistic Force-Free Simulation

2.1. Basic Equations

The basic theories of special relativistic force-free fields were developed by Uchida (1997). We solved induction equations and momentum equations.

\[
\frac{1}{c} \frac{\partial B}{\partial t} + \text{rot} \ E = 0 \tag{2.1}
\]

\[
\frac{\partial P}{\partial t} + \nabla \cdot M = 0 \tag{2.2}
\]

where

\[
P = \frac{1}{4\pi c} E \times B \tag{2.3}
\]

is the Maxwell stress tensor. Here, \( c \) is light speed. (2.1), (2.2), (2.3), and (2.4) is equivalent to the complete set of Maxwell equations with special relativistic force-free conditions \( \rho_e E + \frac{1}{c} J \times B = 0 \). Where \( \rho_e \) is a charge density and \( J \) is a current density. Electric field is given by \( E = - \left( \frac{B^2}{4\pi c} \right)^{-1} P \times B \). We solved equation (2.1) and (2.2) numerically by using HLL method (Harten et al. 1983, Janhunen 2000).

2.2. Initial Conditions

We assumed that a magnetar has dipole magnetic field at the initial state. Twist motion which mimics the crustal motion is given by

\[
v_\phi = \begin{cases} 
0.1c & \theta = 60 - 70 \text{ degree} \ \\
-0.1c & \theta = 110 - 120 \text{ degree} 
\end{cases} \tag{2.5}
\]

as a boundary condition. Here, \( \theta \) denotes azimuthal angle. Rotation of the star is ignored, for simplicity. Number of grids is \((N_r, N_\theta) = (250, 180)\).

2.3. Simulation Results

After magnetic twist is injected from the boundary, fast waves propagate isotropically with speed of light. As Alfvén waves propagate along the magnetic field line, toroidal magnetic field is accumulated inside the magnetic loops. This means that magnetic energy given by twist injection is accumulated inside the loops. When critical twist is accumulated, magnetic loops expand relativistically. The Lorentz factor defined by the drift velocity \( v = c \frac{E \times B}{B^2} \) has two peaks. One is ahead of the expanding magnetic loops and the other is around the front of fast waves. The peak Lorentz factor exceeds 10. Inside the loops, anti-parallel magnetic field creates thin current sheet at the equatorial plane. The dissipation at this current sheet may be responsible for flares (Lyutikov et al. 2002).
Inside the loops, magnetic flux across the equatorial plane is swept up into a shell. Thus strong electric field is induced around the tangential discontinuity where the direction of magnetic field changes from toroidal to poloidal (see Fig.1). Potential energy in observer frame is about

\[ e\phi = \frac{1}{c}(\mathbf{v} \times \mathbf{B})_\phi L \approx 1 \times 10^{18} \left( \frac{v}{c} \right) \left( \frac{B}{10^{10} \text{G}} \right) \left( \frac{L}{100 \text{ km}} \right) \text{eV}. \]  

(2.6)

where \( L \) is the size of the arcade in azimuthal direction. This energy is comparable to the electric potential difference between the footpoints of the arcade. Particle can be accelerated to high energies in fast shocks, tangential discontinuity, and in magnetic loops.

\[ B_x = -B_0 \sin[k_y(y - L_y/2)] \exp(-k_yx) \]  

(3.1)

\[ B_y = B_0 \cos[k_y(y - L_y/2)] \exp(-k_yx) \]  

(3.2)

\[ B_z = 0. \]  

(3.3)

where \( L_y \) is a size of simulation box and \( k_y = 2\pi/L_y \). Periodic boundary condition is adopted in \( y \)-direction. We assume that at the \( x=0 \), the electric field and velocity are given by \( \mathbf{E} = -\frac{1}{c}\mathbf{v} \times \mathbf{B} \), and \( \mathbf{v} = v_0 \sin[2k_y(y - L_y/2)]\hat{z} \), respectively. The sheer velocity \( v_0 \) is assumed to be 0.3c at \( x = 0 \). We assumed electron-positron plasma. Initially, we assume uniform plasma with temperature \( 10^{-2}m_e c^2 \). Number of grid points is \((N_x, N_y) = (2048, 256)\). To avoid the effect of outflowing boundary

§3. Particle Simulations

3.1. Initial Conditions

To study the possibility of particle accelerations in the expanding magnetic loops, we carried out 2-dimensional PIC simulations. We used the magnetic field configuration given by Mikic et al. (1988). Magnetic field is given by

\[ B_x = -B_0 \sin[k_y(y - L_y/2)] \exp(-k_yx) \]  

(3.1)

\[ B_y = B_0 \cos[k_y(y - L_y/2)] \exp(-k_yx) \]  

(3.2)

\[ B_z = 0. \]  

(3.3)

where \( L_y \) is a size of simulation box and \( k_y = 2\pi/L_y \). Periodic boundary condition is adopted in \( y \)-direction. We assume that at the \( x=0 \), the electric field and velocity are given by \( \mathbf{E} = -\frac{1}{c}\mathbf{v} \times \mathbf{B} \), and \( \mathbf{v} = v_0 \sin[2k_y(y - L_y/2)]\hat{z} \), respectively. The sheer velocity \( v_0 \) is assumed to be 0.3c at \( x = 0 \). We assumed electron-positron plasma. Initially, we assume uniform plasma with temperature \( 10^{-2}m_e c^2 \). Number of grid points is \((N_x, N_y) = (2048, 256)\). To avoid the effect of outflowing boundary
condition, we used large simulation area in x-direction. We used about $2.5 \times 10^7$ pairs. The ratio of typical initial magnetic energy density to plasma energy density, $\sigma_0 = B_0^2 / 8\pi n_0 m_e c^2$ is taken to be unity.

3.2. Simulation Results

As magnetic twists are injected, magnetic loops expand in x-directions by enhanced magnetic pressure (Fig.4). By twist injections, particles get energies from the potential energy difference, $e\phi = -e \int (v \times B) dy$. Particles are heated up inside the temperature $\sim 0.2m_e c^2$. This energy is comparable to the electric potential energy difference along the loop. Since the potential energy difference increases with the distance between the footpoints of the magnetic loops, if the distance is $\sim 1$ km, the potential energy difference is about $\sim 10^{20}$ eV. When the potential energy is so large, the temperature of the loops may be limited by the threshold for pair creation, 511 keV (Beloborodov 2006).

![Magnetic field distribution at $t = 480\Omega_g^{-1}$](image1)

Fig. 4. Magnetic field distribution at $t = 480\Omega_g^{-1}$ where $\Omega_g = eB_0 / m_ec$ is the gyro frequency. Color shows $B_z$ and curves show magnetic field lines.

![Energy spectrum of particles](image2)

Fig. 5. Energy spectrum of particles. Solid line shows at time = $80\Omega_g^{-1}$. Dashed line shows at time = $400\Omega_g^{-1}$. Dashed-dotted lines shows thermal spectrum with temperature $=0.2m_e c^2$

§4. Acknowledgement

The authors thank Drs. Hoshino, S., Shibata, M., Hanawa, S., Miyaji for discussion. Numerical computations have been carried out by SR11000 at IMIT, Chiba University, and NEC SX-6 at ISAS/JAXA.

References

10) T. Terasawa et al., Nature 434 1110-1111